Comparison of Control Strategies for Shunt Active Power Filters in Three-Phase Four-Wire Systems

María Isabel Milanés Montero, *Member, IEEE*, Enrique Romero Cadaval, *Member, IEEE*, and Fermín Barrero González, *Member, IEEE*

***Abstract*—Strategies for extracting the three-phase reference currents for shunt active power filters are compared, evaluating their performance under different source and load conditions with the new IEEE Standard 1459 power definitions. The study was applied to a three-phase four-wire system in order to include imbalance.**

Under balanced and sinusoidal voltages, harmonic cancellation and reactive power compensation can be attained in all the methods. However, when the voltages are distorted and/or unbalanced, the compensation capabilities are not equivalent, with some strategies unable to yield an adequate solution when the mains voltages are not ideal. Simulation and experimental results are included.

***Index Terms*—Active filtering, $i_d - i_q$ method, perfect harmonic cancellation method, p–q theory, reference current extraction, unity power factor method (UPF).**

I. INTRODUCTION

POWER electronic converters, ever more widely used in industrial, commercial, and domestic applications, suffer from the problem of drawing nonsinusoidal current and reactive power from the source. This behavior causes voltage distortion that affects other loads connected at the same point of common coupling (PCC). Active power filters (APFs) are being investigated and developed as a viable alternative to solve this problem.

The control strategy for a shunt active power filter (Fig. 1) generates the reference current, i_{Cref}, that must be provided by the power filter to compensate reactive power and harmonic currents demanded by the load. This involves a set of currents in the phase domain, which will be tracked generating the switching signals applied to the electronic converter by means of the appropriate closed-loop switching control technique such as hysteresis or dead-beat control. Sometimes, it is useful to calculate the compensating current in terms of the reference source current ($i_{\mathrm{Cref}} = i_L - i_{\mathrm{Sref}}$).

This paper first presents a review of four control strategies (p–q method, i_d–i_q method, unity power factor (UPF) method, and perfect harmonic cancellation (PHC) method) for the extraction of the reference currents for a shunt active power filter connected to a three-phase four-wire source that supplies a nonlinear load (Fig. 1). Then a comparison of the methods is made by simulations under both ideal and distorted mains voltage conditions and various load conditions. Finally experimental results are presented.

Manuscript received September 27, 2005; revised February 21, 2006. This paper was presented in part at the International Conference on Renewable Energies and Power Quality (ICREPQ'04), Barcelona, Spain, March 31–April 2, 2004. Recommended for publication by Associate Editor F. Peng.

The authors are with the Department of Electronic and Electromechanical Engineering, University of Extremadura, Badajoz S639798, Spain (e-mail: milanes@unex.es).

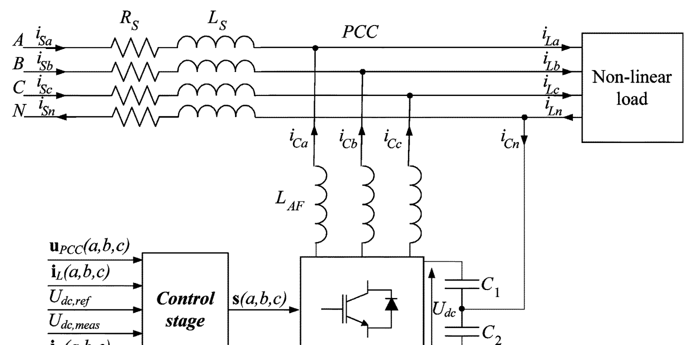


Fig. 1. Three-phase four-wire source with nonlinear load and shunt active power filter.

II. INSTANTANEOUS P–Q STRATEGY

Most APFs have been designed on the basis of instantaneous reactive power theory (or p–q theory) to calculate the desired compensation current. This theory was first proposed by Akagi and co-workers in 1984 [1], and has since been the subject of various interpretations and improvements [2]–[5]. In this method, a set of voltages (u_a, u_b, u_c) and currents (i_a, i_b, i_c) from a three-phase four-wire system are first transformed into a three-axis representation $\alpha-\beta-0$, using the power invariant

$$\begin{bmatrix} u_0 \\ u_\alpha \\ u_\beta \end{bmatrix} = C \begin{bmatrix} u_a \\ u_b \\ u_c \end{bmatrix}; \quad \begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix} = C \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

$$C = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \tag{1}$$

where C is the so called transformation matrix: $\|C\| = 1$; $C^{-1} = C^T$.

The generalized instantaneous active power, p, and instantaneous reactive power, q, defined in [2], [3] in terms of the $\alpha-\beta-0$ components, are given by the following expressions:

$$p = u \cdot i = u_a i_a + u_b i_b + u_c i_c = u_0 i_0 + u_\alpha i_\alpha + u_\beta i_\beta \tag{2}$$

$$q = \begin{bmatrix} q_0 \\ q_\alpha \\ q_\beta \end{bmatrix} \equiv u \times i = \begin{bmatrix} \begin{vmatrix} u_\alpha & u_\beta \\ i_\alpha & i_\beta \end{vmatrix} \\ \begin{vmatrix} u_\beta & u_0 \\ i_\beta & i_0 \end{vmatrix} \\ \begin{vmatrix} u_0 & u_\alpha \\ i_0 & i_\alpha \end{vmatrix} \end{bmatrix}$$

$$q = \|q\| = \sqrt{q_0^2 + q_\alpha^2 + q_\beta^2}. \tag{3}$$

The instantaneous three-phase active power has two components: the instantaneous zero-sequence active power, p_0, and the instantaneous active power due to positive and negative sequence components, $p_{\alpha\beta}$:

$$\begin{aligned} p &= p_0 + p_{\alpha\beta} \\ p_0 &= u_0 i_0 \\ p_{\alpha\beta} &= u_\alpha i_\alpha + u_\beta i_\beta. \end{aligned} \tag{4}$$

Each power component has, in turn, a mean value or dc component and an oscillating value or ac component. For the system shown in Fig. 1, the power components required by the load are:

$$p_L = \overline{p}_L + \tilde{p}_L; \quad q_L = \overline{q}_L + \tilde{q}_L. \tag{5}$$

From (2) and (3), and taking into account that vectors u and q are orthogonal $(u \cdot q = 0)$, the current can be calculated by the inverse transformation

$$\begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix} = \frac{1}{u_0^2 + u_\alpha^2 + u_\beta^2} \begin{bmatrix} u_0 & 0 & u_\beta & -u_\alpha \\ u_\alpha & -u_\beta & 0 & u_0 \\ u_\beta & u_\alpha & -u_0 & 0 \end{bmatrix} \begin{bmatrix} p \\ q_0 \\ q_\alpha \\ q_\beta \end{bmatrix}. \tag{6}$$

The objective of the p–q strategy is to get the source to give only the constant active power demanded by the load, $p_S = \overline{p}_{L\alpha\beta} + \overline{p}_{L0}$. In addition, the source must deliver no zero-sequence active power, $i_{S0ref} = 0$ (so that the zero-sequence component of the voltage at the PCC does not contribute to the source power). The reference source current in the $\alpha - \beta - 0$ frame is therefore

$$\begin{aligned} \begin{bmatrix} i_{S0_{\text{ref}}} \\ i_{S\alpha_{\text{ref}}} \\ i_{S\beta_{\text{ref}}} \end{bmatrix} &= \frac{1}{u_\alpha^2 + u_\beta^2} \begin{bmatrix} 0 & 0 & u_\beta & -u_\alpha \\ u_\alpha & -u_\beta & 0 & u_0 \\ u_\beta & u_\alpha & -u_0 & 0 \end{bmatrix} \\ &\quad \times \begin{bmatrix} \overline{p}_{L\alpha\beta} + \overline{p}_{L0} \\ 0 \\ 0 \\ 0 \end{bmatrix} \\ &= \frac{\overline{p}_{L\alpha\beta} + \overline{p}_{L0}}{u_\alpha^2 + u_\beta^2} \begin{bmatrix} 0 \\ u_\alpha \\ u_\beta \end{bmatrix} \end{aligned} \tag{7}$$

where the vector u is the voltage at the PCC.

III. $I_D - I_Q$ Method

This method is also known as synchronous reference frame (SRF) [6], [7]. Here, the reference frame d–q (d direct axis, q quadrature axis) is determined by the angle θ with respect to the α–β frame used in the p–q theory. The transformation from α–β–0 frame to d–q–0 frame is given by

$$\begin{bmatrix} i_0 \\ i_d \\ i_q \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix}. \tag{8}$$

If the d axis is in the direction of the voltage space vector, since the zero-sequence component is invariant, the transformation is given by

$$\begin{aligned} \begin{bmatrix} i_d \\ i_q \end{bmatrix} &= S \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} \\ S &= \frac{1}{\sqrt{u_\alpha^2 + u_\beta^2}} \begin{bmatrix} u_\alpha & u_\beta \\ -u_\beta & u_\alpha \end{bmatrix} \end{aligned} \tag{9}$$

where the transformation matrix, S, satisfies: $\|S\| = 1$; $S^{-1} = S^T$.

Each current component (i_d, i_q) has an average value or dc component and an oscillating value or ac component

$$\begin{aligned} i_d &= \overline{i}_d + \tilde{i}_d \\ i_q &= \overline{i}_d + \tilde{i}_q. \end{aligned} \tag{10}$$

The compensating strategy (for harmonic reduction and reactive power compensation) assumes that the source must only deliver the mean value of the direct-axis component of the load current. The reference source current will therefore be

$$i_{Sd_{\text{ref}}} = \overline{i}_{Ld}; \quad i_{Sq_{\text{ref}}} = i_{S0_{\text{ref}}} = 0. \tag{11}$$

From (9), the direct-axis component of the load current is

$$i_{Ld} = \frac{u_\alpha \cdot i_{L\alpha} + u_\beta \cdot i_{L\beta}}{\sqrt{u_\alpha^2 + u_\beta^2}} = \frac{p_{L\alpha\beta}}{\sqrt{u_\alpha^2 + u_\beta^2}} = \overline{i}_{Ld} + \tilde{i}_{Ld}. \tag{12}$$

The dc component of the above equation will be

$$\overline{i}_{Ld} = \left(\frac{p_{L\alpha\beta}}{\sqrt{u_\alpha^2 + u_\beta^2}} \right)_{\text{dc}} \tag{13}$$

where the subscript "dc" is to be understood as the mean value of the expression within parentheses.

The reference source current must be in phase with the voltage at the PCC but with no zero-sequence component. It will therefore be obtained in the α–β–0 frame by multiplying (13) by a unit vector in the direction of the PCC voltage space vector, excluding the zero-sequence component

$$\begin{aligned} i_{S_{\text{ref}}} &= \overline{i}_{Ld} \frac{1}{\sqrt{u_\alpha^2 + u_\beta^2}} \begin{bmatrix} 0 \\ u_\alpha \\ u_\beta \end{bmatrix} \\ &= \left(\frac{p_{L\alpha\beta}}{\sqrt{u_\alpha^2 + u_\beta^2}} \right)_{\text{dc}} \frac{1}{\sqrt{u_\alpha^2 + u_\beta^2}} \begin{bmatrix} 0 \\ u_\alpha \\ u_\beta \end{bmatrix}. \end{aligned} \tag{14}$$

IV. UPF Strategy

The compensating strategy known as the unity power factor (UPF) method has the objective that the load plus the compensator must be viewed by the source as a resistance [8], [9]. This method is also known as the "voltage synchronization method" because the source current space vector is desired to be in phase with the PCC voltage space vector

$$i_{S_{\text{ref}}} = K \cdot u \tag{15}$$

where K is a constant whose value depends on the PCC voltage and the load.

The power delivered by the source will be

$$p_S = u \cdot i_S = u \cdot K \cdot u = K\left(u_0^2 + u_\alpha^2 + u_\beta^2\right). \tag{16}$$

The conductance K can be determined with the criterion that the power delivered by the source equals the dc component of the instantaneous active power of the load, so that

$$K = \frac{\overline{p}_{L\alpha\beta} + \overline{p}_{L0}}{\left(u_0^2 + u_\alpha^2 + u_\beta^2\right)_{\text{dc}}}. \tag{17}$$

Finally, the reference source current will be given by

$$\begin{bmatrix} i_{S0_{\text{ref}}} \\ i_{S\alpha_{\text{ref}}} \\ i_{S\beta_{\text{ref}}} \end{bmatrix} = K \begin{bmatrix} u_0 \\ u_\alpha \\ u_\beta \end{bmatrix} = \frac{\overline{p}_{L\alpha\beta} + \overline{p}_{L0}}{\left(u_0^2 + u_\alpha^2 + u_\beta^2\right)_{\text{dc}}} \begin{bmatrix} u_0 \\ u_\alpha \\ u_\beta \end{bmatrix}. \tag{18}$$

V. PHC Strategy

The perfect harmonic cancellation (PHC) method can be regarded as a modification of the three previous theories. Its objective is to compensate all the harmonic currents and the fundamental reactive power demanded by the load in addition to eliminating the imbalance. The source current will therefore be in phase with the fundamental positive-sequence component of the voltage at the PCC [8].

The reference source current will be given by

$$i_{Sref} = K \cdot u_1^+ \tag{19}$$

where u_1^+ is the PCC voltage space vector with a single fundamental positive-sequence component.

The power delivered by the source will then be

$$p_S = u \cdot i_{S_{\text{ref}}} = u \cdot K \cdot u_1^+ = K\left(u_\alpha u_{\alpha 1}^+ + u_\beta u_{\beta 1}^+\right). \tag{20}$$

The constant K will be determined with the condition that the above source power equals the dc component of the instantaneous active power demanded by the load

$$K = \frac{\overline{p}_{L\alpha\beta} + \overline{p}_{L0}}{u_{\alpha 1}^{+2} + u_{\beta 1}^{+2}}. \tag{21}$$

Finally, the reference source current will be given by

$$\begin{bmatrix} i_{S0_{\text{ref}}} \\ i_{S\alpha_{\text{ref}}} \\ i_{S\beta_{\text{ref}}} \end{bmatrix} = K \begin{bmatrix} 0 \\ u_{\alpha 1} \\ u_{\beta 1} \end{bmatrix} = \frac{\overline{p}_{L\alpha\beta} + \overline{p}_{L0}}{u_{\alpha 1}^{+2} + u_{\beta 1}^{+2}} \begin{bmatrix} 0 \\ u_{\alpha 1}^+ \\ u_{\beta 1}^+ \end{bmatrix}. \tag{22}$$

VI. Comparative Evaluation. Simulation Results

Table I summarizes the expressions for determining the reference source current in the four compensation strategies. One can notice that some of these expressions can be obtained from the instantaneous active current $i_P(t)$ proposed in [10]

$$i_P(t) = \frac{P_L(t)}{V_P^2(t)} v_P(t) \tag{23}$$

TABLE I
Expressions for the Reference Source Currents

Control strategy	Reference Source Current
UPF strategy	$\begin{bmatrix} i_{S0_{ref}} \\ i_{S\alpha_{ref}} \\ i_{S\beta_{ref}} \end{bmatrix} = \frac{\overline{p}_{L\alpha\beta} + \overline{p}_{L0}}{\left(u_0^2 + u_\alpha^2 + u_\beta^2\right)_{dc}} \begin{bmatrix} u_0 \\ u_\alpha \\ u_\beta \end{bmatrix}$
Generalized p-q strategy	$\begin{bmatrix} i_{S0_{ref}} \\ i_{S\alpha_{ref}} \\ i_{S\beta_{ref}} \end{bmatrix} = \frac{\overline{p}_{L\alpha\beta} + \overline{p}_{L0}}{u_\alpha^2 + u_\beta^2} \begin{bmatrix} 0 \\ u_\alpha \\ u_\beta \end{bmatrix}$
i_d-i_q strategy	$\begin{bmatrix} i_{S0_{ref}} \\ i_{S\alpha_{ref}} \\ i_{S\beta_{ref}} \end{bmatrix} = \left(\frac{p_{L\alpha\beta}}{\sqrt{u_\alpha^2 + u_\beta^2}}\right)_{dc} \frac{1}{\sqrt{u_\alpha^2 + u_\beta^2}} \begin{bmatrix} 0 \\ u_\alpha \\ u_\beta \end{bmatrix}$
PHC strategy	$\begin{bmatrix} i_{S0_{ref}} \\ i_{S\alpha_{ref}} \\ i_{S\beta_{ref}} \end{bmatrix} = \frac{\overline{p}_{L\alpha\beta} + \overline{p}_{L0}}{u_{\alpha 1}^{+2} + u_{\beta 1}^{+2}} \begin{bmatrix} 0 \\ u_{\alpha 1}^+ \\ u_{\beta 1}^+ \end{bmatrix}$

where $v_P(t)$ is the reference voltage, $V_P(t)$ its RMS value and $P_L(t)$ the average load active power, both calculated in the averaging interval T_C. For a three-phase system, (23) can be expressed in the vector form used in this paper as

$$i_{\text{Sref}} = i_P = \frac{\overline{p}_L}{\left(v_{P0}^2 + v_{P\alpha}^2 + v_{P\beta}^2\right)_{\text{dc}}} \begin{bmatrix} v_{P0} \\ v_{P\alpha} \\ v_{P\beta} \end{bmatrix}. \tag{24}$$

By changing T_C and v_P, different compensation objectives can be attained [11]: if T_C is the fundamental period, T_1, and $v_P = u_{\text{PCC}}$, (24) equals the reference source current proposed by UPF, while selecting $v_P = u_{\text{PCC1}}^+$ the PHC reference source current is obtained.

For comparison, various simulations were conducted with both ideal and distorted mains voltage and under different load current conditions. In all cases, the phase angle between the fundamental components of source voltage and load current was 30° inductive.

The figures that follow are based on normalized quantities. For balanced cases the phase a magnitudes, and for unbalanced cases all three phase magnitudes, will be shown. For all cases, the information is organized as follows.

a) Waveform and frequency spectrum: u_{PCC}, i_L, $i_{S-\text{UPF}}$, i_{S-pq}, $i_{S-id-iq}$, $i_{S-\text{PHC}}$.
b) (b) Instantaneous powers (left: thick-$p_{\alpha\beta}$, thin-q; right: p_0): Load, source UPF, source p–q, source i_d–i_q, source PHC.

The terms relating to power concepts in the tables are based on the new definitions of power proposed by the IEEE Working Group on Nonsinusoidal Situations with the modifications suggested by Depenbrock [12], [13] as collected in IEEE Standard

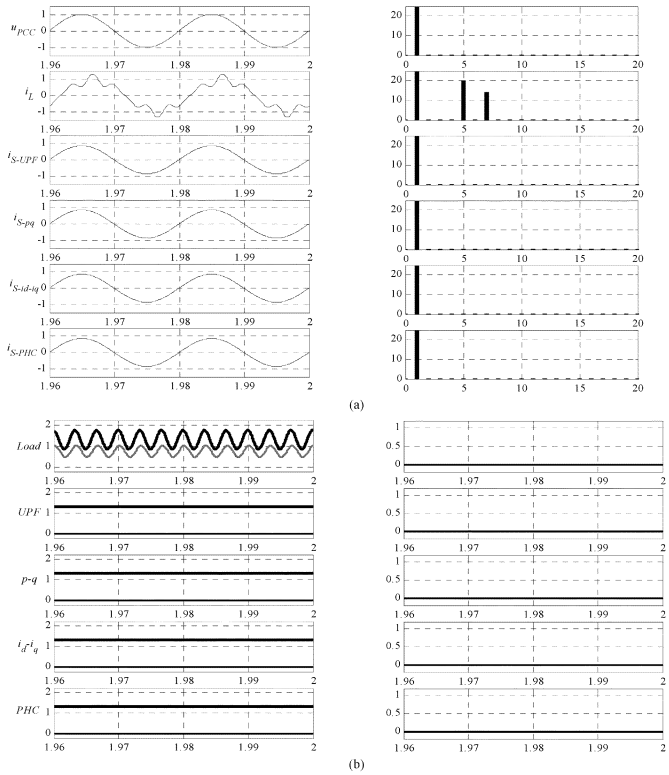


Fig. 2. Simulation results for case A.

1459 [14]. For a three-phase four-wire system, the equivalent voltage, current, and apparent power are given by

$$U_e = \sqrt{\frac{U_a^2 + U_b^2 + U_c^2}{3}}; \quad I_e = \sqrt{\frac{I_a^2 + I_b^2 + I_c^2 + I_n^2}{3}}$$
$$S_e = 3U_e I_e. \tag{25}$$

The total active power is obtained by adding the active power in each phase

$$P = \sum_k \sum_h U_{kh} I_{kh} \cos\varphi_{kh} \tag{26}$$

where k means the phase (a, b, or c), h is the order of the harmonic and φ_{kh} is the angle between the hth harmonic voltage and the hth harmonic current for phase k. The total power factor is therefore

$$\mathrm{PF} = \frac{P}{S_e}. \tag{27}$$

A. Case A: Ideal Mains Voltage: Balanced and Distorted (Fifth and Seventh Harmonics) Load Current

Simulation results for case A are shown in Fig. 2 and summarized in Table II. Both source voltage and current are sinusoidal and in phase. Hence, reactive power and harmonics are fully compensated. The source supplies only the constant power demanded by the load. With ideal mains voltage, therefore, all the strategies are equivalent.

TABLE II
SUMMARY OF SIMULATION RESULTS FOR CASE A

	i_L	$i_{S\text{-}UPF}$	$i_{S\text{-}p\text{-}q}$	$i_{S\text{-}id\text{-}iq}$	$i_{S\text{-}PHC}$
THD	24.46%	0.041%	0.041%	0.041%	0.044%
I_1	0.707	0.612	0.612	0.612	0.612
I	0.728	0.612	0.612	0.612	0.612
dPF	0.866	1	1	1	1
P	1.299	1.299	1.299	1.299	1.299
S_e	1.544	1.299	1.299	1.299	1.299
PF	0.841	1	1	1	1

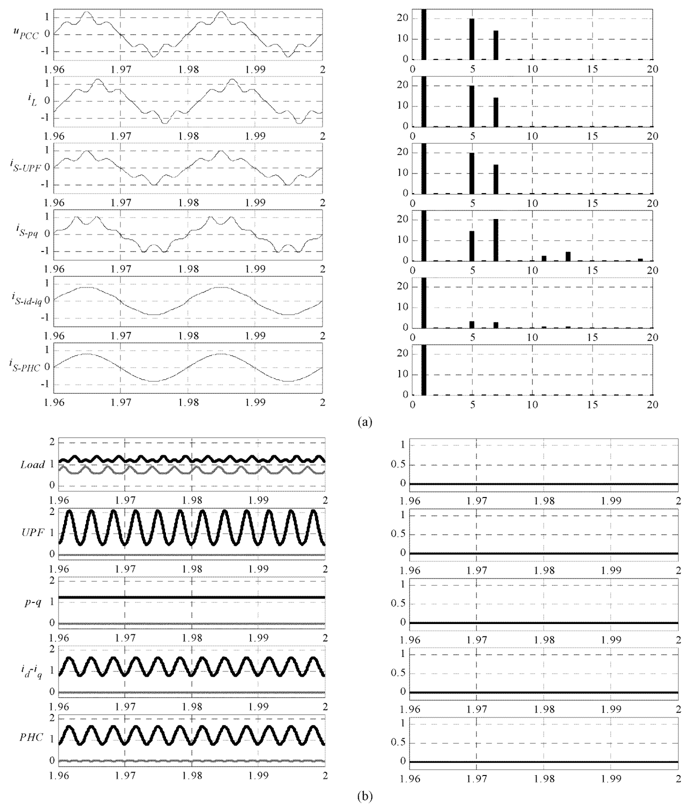


Fig. 3. Simulation results for case *B*.

TABLE III
SUMMARY OF SIMULATION RESULTS FOR CASE *B*

	i_L	$i_{S\text{-}UPF}$	$i_{S\text{-}p\text{-}q}$	$i_{S\text{-}ip\text{-}iq}$	$i_{S\text{-}PHC}$
THD	24.46%	24.66%	25.79%	4.23%	0.045%
I_1	0.707	0.543	0.611	0.610	0.575
I	0.728	0.559	0.631	0.611	0.575
dPF	0.866	1	1	1	1
P	1.221	1.221	1.221	1.221	1.221
S_e	1.590	1.221	1.379	1.256	1.257
PF	0.768	1	0.885	0.972	0.971

B. Case B: Balanced and Distorted (Fifth and Seventh Harmonics) Source Voltages; Balanced and Distorted (Fifth and Seventh Harmonics) Load Currents

Simulation results for case B are presented in Fig. 3 and Table III. Comparing the frequency spectra, one observes that only the PHC strategy cancels all the harmonics in the source current. The UPF strategy maintains the source voltage total harmonic distortion (THD), whereas the p–q strategy even increases this ratio because it contains new harmonics at frequencies not present in the load currents. One could conclude from Table III that PHC and i_d–i_q are able to satisfy the IEEE-519 Standard harmonic current limits [15].

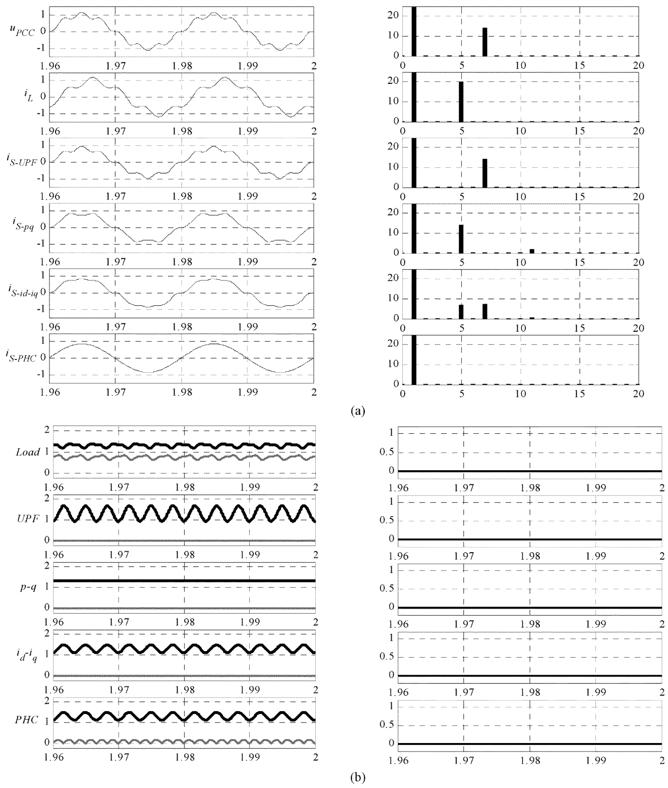


Fig. 4. Simulation results for case C.

TABLE IV
SUMMARY OF SIMULATION RESULTS FOR CASE C

	i_L	$i_{S\text{-}UPF}$	$i_{S\text{-}p\text{-}q}$	$i_{S\text{-}id\text{-}iq}$	$i_{S\text{-}PHC}$
THD	19.887%	14.317%	14.429%	10.160%	0.03%
I_1	0.707	0.600	0.612	0.606	0.612
I	0.721	0.606	0.619	0.609	0.612
dPF	0.866	1	1	1	1
P	1.299	1.299	1.299	1.299	1.299
S_e	1.545	1.299	1.326	1.306	1.312
PF	0.841	1	0.980	0.995	0.990

In terms of power, PHC is the only strategy which does not fully compensate the instantaneous reactive power, as can be observed in Fig. 3(b). However, with the new definition of PF indicated in (27), only UPF attain a unity value for this index, as is seen in Table III. All the strategies correctly compensate the fundamental reactive power, yielding a unity displacement power factor (dPF).

C. Case C: Balanced and Distorted (Seventh Harmonic) Source Voltage; Balanced and Distorted (Fifth Harmonic) Load Current

Simulation results for case C are presented in Fig. 4 and Table IV. These results are similar to case B, indicating that the i_d–i_q strategy does not satisfy IEEE-519. This is because it generates nonsinusoidal reference source currents due to the different harmonics orders in source voltages and load currents.

D. Case D: Unbalanced and Undistorted Source Voltages ($U^-_{\mathrm{PCCe1}}/U^+_{\mathrm{PCCe1}} = U^0_{\mathrm{PCCe1}}/U^+_{\mathrm{PCCe1}}$ = 23.1%); Balanced and Undistorted Load Currents

Simulation results for case D are presented in Fig. 5 and Table V. One observes that only PHC produces balanced source

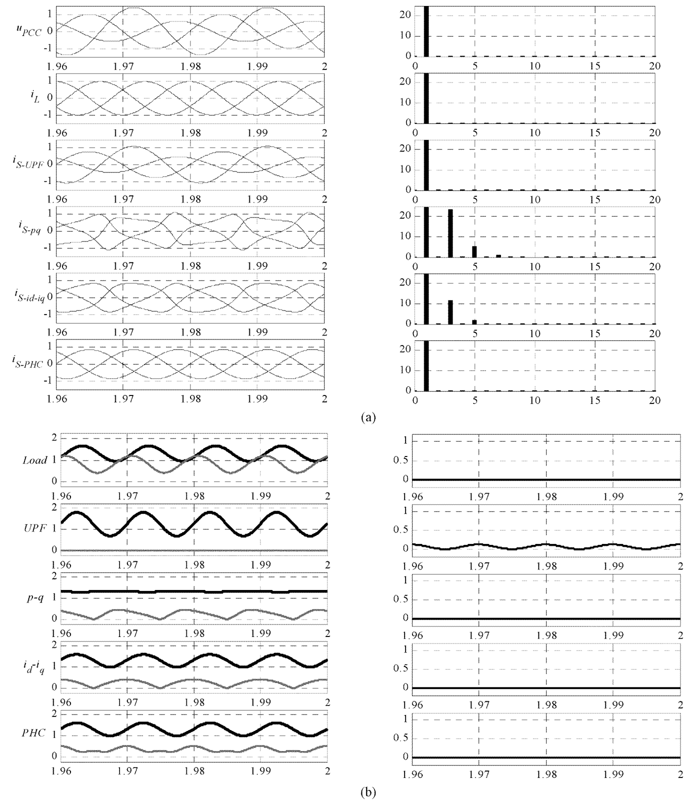


Fig. 5. Simulation results for case D.

TABLE V
SUMMARY OF SIMULATION RESULTS FOR CASE D

	i_L	$i_{S\text{-}UPF}$	$i_{S\text{-}p\text{-}q}$	$i_{S\text{-}id\text{-}iq}$	$i_{S\text{-}PHC}$
I_a	0.707	0.552	0.630	0.604	0.612
THD	0%	0.219%	23.96%	11.6%	0.224%
I_b	0.707	0.776	0.629	0.662	0.611
THD	0%	0%	23.76%	10.67%	0%
I_c	0.707	0.332	0.631	0.541	0.613
THD	0%	0%	23.77%	11.94%	0%
P	1.299	1.299	1.299	1.299	1.299
S_e	1.578	1.390	1.406	1.349	1.366
PF	0.823	0.935	0.924	0.963	0.951

TABLE VI
SUMMARY OF SIMULATION RESULTS FOR CASE E

	i_L	$i_{S\text{-}UPF}$	$i_{S\text{-}p\text{-}q}$	$i_{S\text{-}id\text{-}iq}$	$i_{S\text{-}PHC}$
I_a	0.707	0.611	0.888	0.616	0.754
THD	0%	0.28%	61.69%	26.19%	0.38%
I_b	0.990	0.858	0.887	0.683	0.752
THD	0%	0%	61.72%	24.88%	0%
I_c	0	0	0.893	0.414	0.759
THD	-	-	61.23%	32.28%	0%
P	1.28	1.28	1.28	1.05	1.28
S_e	1.83	1.58	1.87	1.23	1.59
PF	0.701	0.809	0.684	0.860	0.803

currents. The frequency spectrum in Fig. 5 corresponds to phase a. However, THD values for all the phases are presented in Table V. Although the voltage and load currents are sinusoidal, the p–q and i_d–i_q strategies yield source currents with harmonics. With the UPF strategy, the source delivers zero-sequence active power although this power term is not demanded by the load, leading to a PF as calculated from (27) of less than unity.

TABLE VII
PARAMETERS OF THE EXPERIMENTAL PROTOTYPE

$S_B^{3\phi}$ (VA)	$U_B^{L\text{-}L}$ (V)	L_{AF} (mH)	$C_1 = C_2$ (mF)	$U_{dc,ref}$ (V)	R_L (Ω)
1200	400 V	30	1	900	440

E. Case E: Unbalanced and Undistorted Source Voltages ($U^-_{\text{PCCe1}}/U^+_{\text{PCCe1}} = U^0_{\text{PCCe1}}/U^+_{\text{PCCe1}}$ =52%); Unbalanced and Undistorted Load Currents ($I^-_{Le1}/I^+_{Le1} = I^0_{Le1}/I^+_{Le1}$ =52%)

Simulation results for case E are presented in Fig. 6 and Table VI. The frequency spectra of phase c are shown since this is the least favourable phase. Here, in contrast to case D, there is zero-sequence instantaneous active power demanded by the load. Only the *p–q* and PHC strategies can eliminate this power term, while UPF maintains the zero-sequence component of the voltage in the current (yielding a PF<1), and i_d–i_q is unable to compensate this term, as can be concluded from Table I (the term p_{L0} is not taken into account for extracting the reference current, but $i_{S0-id-iq}$ =0, so $p_{S0-id-iq}$ =0) and the active power data in Table VI (P_L = 1.28 W while P_S = 1.05 W because the dc zero-sequence active power demanded by the load is not delivered by the source, what implies the need of an external source to provide this power). In terms of the distortion and imbalance, the results are similar to case D.

F. Analysis of the Simulation Results

From the above figures and tables, one may draw the following conclusions:

UPF: The source current waveforms will be identical to the voltage waveforms and can thus not comply with the IEEE Standard 519 limits, or will be unbalanced depending on the voltage. The instantaneous reactive power demanded by the load is fully eliminated [q_S calculated from (3) and (15) is null in all cases, as is seen in Figs. 2(b)–6(b)]. In three-phase four-wire systems with zero-sequence components in the voltage at the PCC (cases D and E), the energy transfer is not maximal, yielding a power factor less than unity and source currents with greater RMS values. Furthermore, in these situations the source delivers zero-sequence power—Figs. 5(b) and 6(b)—even though the load does not demand this power term (case D).

p–q: The instantaneous active power delivered by the source equals the constant active load power ($p_S = p_{S\alpha\beta} = \overline{p}_{L\alpha\beta} + \overline{p}_{L0}$), as can be observed in Figs. 2(b)–6(b). The generalized *p–q* strategy has disadvantages when the voltage at the PCC has harmonics and/or is unbalanced. In these situations the modulus of the instantaneous vector of the PCC voltage with no zero-sequence component, $u_{\alpha\beta,}$ is not constant, so that, as follows from Table I, the reference current is obtained by multiplying a time-varying term by the vector $u_{\alpha\beta,}$. This could even include harmonics of orders not contained in the load current [8], as is seen in the frequency spectra of Figs. 3(a)–6(a).

Although the original and modified *p–q* theories have been the most extensively used strategies for conditioner control, and have been a benchmark in the development of new methods, they constituted the strategy most sensitive to harmonics and imbalance in the mains voltages. In the present simulations, this method gave the poorest results in terms of THD and PF, and worked adequately only in the case of ideal mains voltages.

i_d–i_q: In this method, the source delivers the dc direct load current component. However, this technique introduces many errors when the PCC voltage contains harmonics or imbalance due to negative-sequence components because the unit vector in the direction of the vector $u_{\alpha\beta}$ is not calculated correctly—see (14). This is the reason for the harmonics in the source current in cases B–E. Another drawback, as was mentioned above, is that the method is

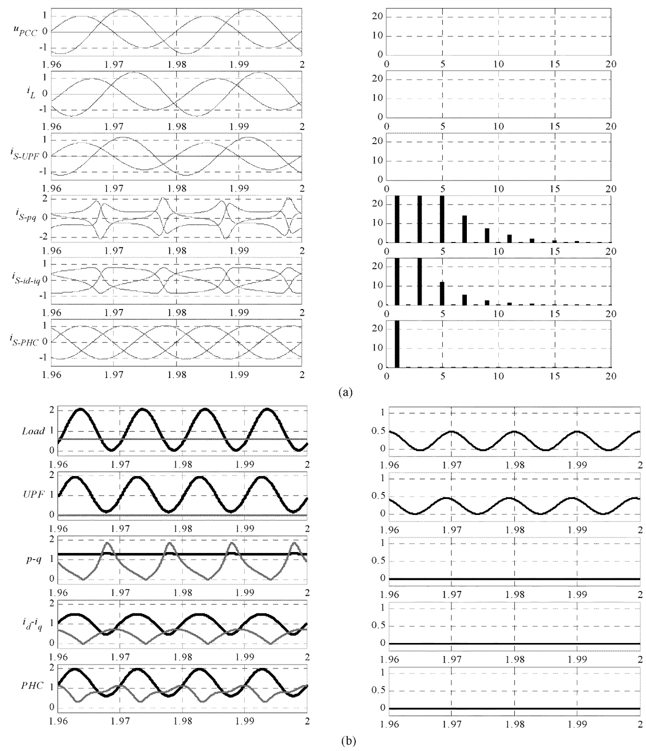


Fig. 6. Simulation results for case E.

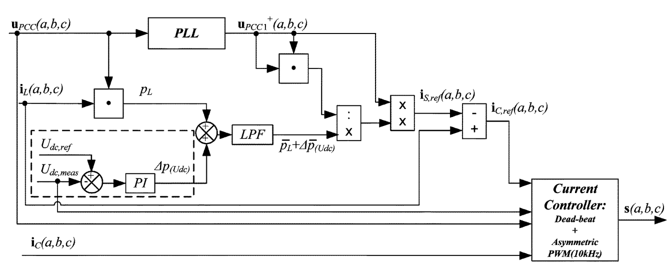
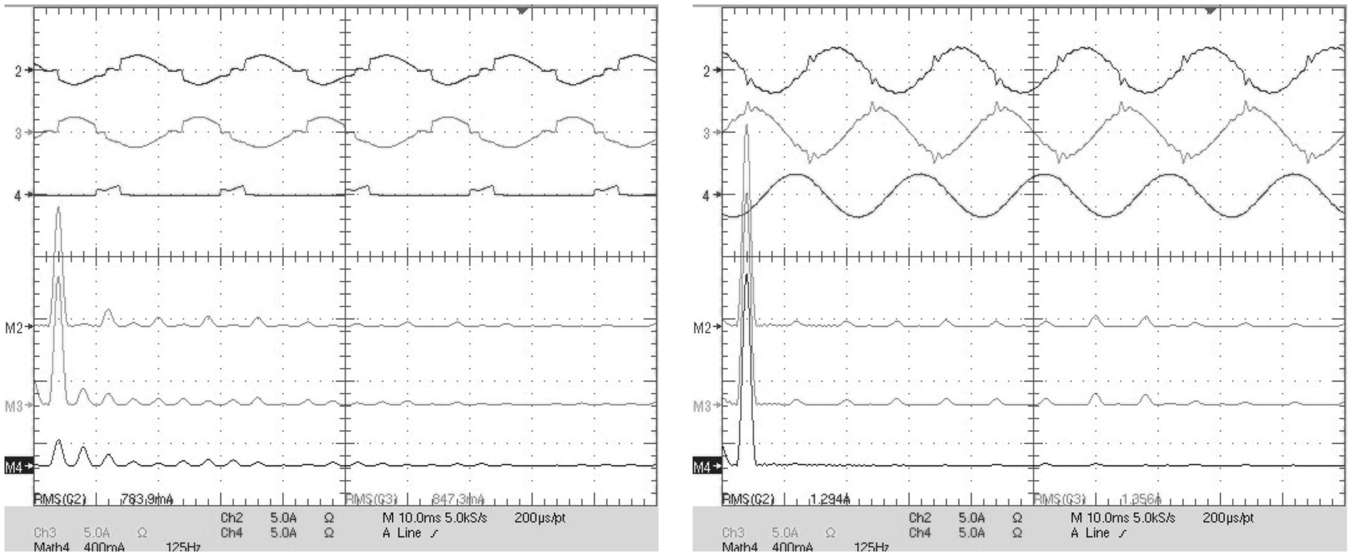


Fig. 7. Control stage scheme formed by the PHC control strategy block and the current controller.



(a) (b)

Fig. 8. Experimental results under conditions of unbalanced source voltage with zero-sequence component. Waveforms (5 A/division) and frequency spectra (400 mA/division) for phases a, b, and c: (a) load currents and (b) source currents after compensation.

unable to compensate the dc zero-sequence active power demanded by the load, so that an external source would be needed in such situations. This problem could be obviated if (14) were replaced by

$$\begin{bmatrix} i_{S0_{\mathrm{ref}}} \\ i_{S\alpha_{\mathrm{ref}}} \\ i_{S\beta_{\mathrm{ref}}} \end{bmatrix} = \left(\frac{p_{L\alpha\beta} + p_{L0}}{\sqrt{u_\alpha^2 + u_\beta^2}} \right)_{\mathrm{dc}} \frac{1}{\sqrt{u_\alpha^2 + u_\beta^2}} \begin{bmatrix} 0 \\ u_\alpha \\ u_\beta \end{bmatrix}. \quad (28)$$

PHC: This strategy ensures sinusoidal and balanced currents in phase with positive fundamental harmonic voltages, although harmonics and/or imbalance appear in the PCC voltage. However, the source delivers reactive power and ac components of active power—Figs. 3(b)–6(b)—so that PF is less than unity.

VII. Experimental Results

The simulation analysis showed the least favourable situation to correspond to the case of a three-phase four-wire system with zero-sequence components in the voltage at the PCC. In these situations (cases D and E in Section VI), only the PHC strategy has the capability to eliminate imbalance in the source currents.

To test this conclusion, experiments were conducted with a 1.2-kVA laboratory prototype APF. The experimental arrangement was similar to that of Fig. 1, where the nonlinear load was a three-phase controlled rectifier with resistive load, R_L. Parameters of the experimental prototype are summarized in Table VII. The converter was a neutral-pointed-clamped VSI, operating with the PHC strategy. The control strategy scheme is shown in Fig. 7, where the typical loop for controlling the dc bus voltage was added. A dead-beat current technique was used, and the switching signals were generated employing asymmetric PWM with a 10-kHz switching frequency. The current controller block is also presented in Fig. 7.

Fig. 8 shows the experimental results when the phase c source conductor was connected to the neutral source conductor, forcing negative-sequence and zero-sequence components ($U^{-}_{\mathrm{PCCe1}}/U^{+}_{\mathrm{PCCe1}} = U^{0}_{\mathrm{PCCe1}}/U^{+}_{\mathrm{PCCe1}}$ =50%) to appear in the voltage at the PCC. Fig. 8(a) shows the waveforms and frequency spectra of the load or source currents before compensation. After compensation, Fig. 8(b), the source currents were sinusoidal and balanced, even though the conditions of use were so unfavourable.

VIII. Conclusion

This paper has provided a comparative analysis of four control strategies for shunt APFs installed in three-phase four-wire systems with harmonic distortion and/or imbalance. It was shown that the p–q strategy (maybe the most widely used) and the i_d–i_q strategy are the most sensitive to distortion and imbalance in the voltages at the PCC.

Although the objective of UPF is to attain unity PF and to minimize the source current RMS values, with the new power definitions of IEEE Standard 1459 these goals are not achieved in the case of three-phase four-wire systems with zero-sequence components in the voltage.

The simulations showed that, if one seeks compliance with harmonics standards, imbalance elimination, and reactive power compensation, PHC is the only strategy which is capable of correct action under any conditions of use. This was confirmed by experiment in the case of the least favourable situation.

References

[1] H. Akagi, Y. Kanazawa, and A. Nabae, "Instantaneous reactive power compensators comprising switching devices without energy storage components," *IEEE Trans. Ind. Appl.*, vol. IA-20, no. 3, pp. 625–630, May/Jun. 1984.

[2] F.-Z. Peng, G. W. Ott, and D. J. Adams, "Harmonic and reactive power compensation based on the generalized instantaneous reactive power theory for three-phase four-wire systems," *IEEE Trans. Power Electron.*, vol. 13, no. 6, pp. 1174–1181, Nov. 1998.

[3] H. Akagi, S. Ogasawara, and H. Kim, “The theory of instantaneous power in three-phase four-wire systems: a comprehensive approach,” in *Proc. IEEE IAS Annu. Meeting*, 1999, pp. 431–439.
[4] J. Afonso, C. Couto, and J. Martins, “Active filters with control based on the p–q theory,” *IEEE Ind. Electron. Soc. Newslett.*, pp. 5–11, Sep. 2000.
[5] M. Aredes and E. H. Watanabe, “New control algorithms for series and shunt three-phase four-wire active power filters,” *IEEE Trans. Power Delivery*, vol. 11, no. 3, pp. 1649–1656, Jul. 1995.
[6] A. Nabae and T. Tanaka, “A new definition of instantaneous active-reactive current and a power based on instantaneous space vectors on polar coordinates in three phase circuits,” *IEEE Trans. Power Delivery*, vol. 11, no. 3, pp. 1238–1243, Jul. 1996.
[7] V. Soares, P. Verdelho, and G. D. Marques, “An instantaneous active and reactive current component method for active filters,” *IEEE Trans. Power Electron.*, vol. 15, no. 4, pp. 660–669, Jul. 2000.
[8] M.-R. Rafiei, H. A. Toliyat, R. Ghazi, and T. Gopalarathanam, “An optimal and flexible control strategy for active filtering and power factor correction under nonsinusoidal line voltages,” *IEEE Trans. Power Delivery*, vol. 16, no. 2, pp. 297–305, Apr. 2001.
[9] A. Cavallani and G. C. Montarani, “Compensation strategies for shunt active-filter control,” *IEEE Trans. Power Electron.*, vol. 9, no. 6, pp. 587–593, Nov. 1994.
[10] Y. Xu, L. M. Tolbert, F. Z. Peng, J. N. Chiasson, and J. Chen, “Compensation-based nonactive power definition,” *IEEE Power Electron. Lett.*, vol. 1, no. 2, pp. 455–450, Jun. 2003.
[11] Y. Xu, L. M. Tolbert, J. N. Chiasson, and F. Z. Peng, “Dynamic response of active filter using a generalized nonactive power theory,” in *Proc. IEEE IAS Annu. Meeting*, Oct. 2005, pp. 1225–1231.
[12] M. Depenbrock, V. Staudt, and H. Wrede, “A theoretical investigation of original and modified instantaneous power theory applied to four-wire systems,” *IEEE Trans. Ind. Appl.*, vol. 39, no. 4, pp. 1160–1168, Jul./Aug. 2003.
[13] IEEE Working Group on Nonsinusoidal Situations: Efects on Meter Performance and Definitions of Power, “Practical definitions for powers in systems with nonsinusoidal waveforms and unbalanced loads: a discussion,” *IEEE Trans. Power Delivery*, vol. 11, no. 1, pp. 79–101, Jan. 1996.
[14] A. E. Emanuel, “Summary of IEEE Standard 1459: definitions for the measurement of electric power quantities under sinusoidal, nonsinusoidal, balanced, or unbalanced conditions,” *IEEE Trans. Ind. Appl.*, vol. 40, no. 3, pp. 869–876, May/Jun. 2004.
[15] *IEEE Recommended Practices and Requirements for Harmonic Control in Electrical Power Systems*, IEEE Std. 519-1992, 1992.

María Isabel Milanés Montero (S’03–M’06) recieved the M.Sc. degree in industrial engineering and the Ph.D. degree in power conditioners applied to improve power quality from the University of Extremadura, Badajoz, Spain, in 1997 and 2005, respectively.

In November 1998, she joined the Electronic and Electromechanical Engineering Department, School of Industrial Engineering, University of Extremadura, as an Assistant Professor. Her major fields of interest include solid-state power converter design and control, electromagnetic interferences, power quality, renewable energy sources control, and electrical machine drives.

Enrique Romero Cadaval (S’03–M’05) received the M.Sc. degree in electronic industrial engineering from ICAI, Universidad Pontificia de Comillas, Madrid, Spain, in 1992 and the Ph. D. degree from the Universidad de Extremadura, Badajoz, Spain, in 2004.

He is a Full Professor in power electronics at the Universidad de Extremadura. His research interests are power electronics in the power system, power quality, electromagnetic interferences, active power filters, and renewable energy sources control.

Fermín Barrero González (M’95) received the M.Sc. degree in electrical engineering from the Universidad Politécnica de Madrid, Madrid, Spain, in 1984 and the Ph. D. degree from the Universidad Nacional de Educación a Distancia, Madrid, in 1995.

He is a Full Professor in electrical engineering at the Universidad de Extremadura, Badajoz, Spain. His research interests are power electronics in the power system, FACTS, active power filters, and electrical machine drives.

Dr. Barrero is a member of the IEEE IAS Industrial Static Converters Committee, European Working Group.